Abstract— In this paper, two types of distributed constant false alarm rate (CFAR) detectors; binary and fuzzy distributed detectors, are introduced. In these two types of distributed detectors, it was assumed that the clutter parameters at the local sensors are unknown and each local detector performs CFAR processing based on maximum likelihood (ML) and order statistic (OS) CFAR processor before transmitting data to the fusion center. At the fusion center, received data are weighted by a binary or a fuzzy weighting function, and combined according to deterministic rules, constructing global test statistics. In the binary and fuzzy types, we consider the various distributed detectors based on binary and fuzzy rules used in fusion center and CFAR detector used in local detectors. The performance of the two types of distributed detectors are analysed and compared with each other. The simulation results indicate the superiority and robust performance of fuzzy type in homogenous and non-homogenous situations.

I. INTRODUCTION

Distributed detection schemes are needed when system performance factor such as speed, reliability against failure of individual receivers, survivability, increase in the number of targets under consideration, and constraint over the communication bandwidth are taken into account. When the background clutter is non-stationary, adaptive threshold techniques are required in order to maintain a nearly constant false alarm rate (CFAR). A variety of CFAR techniques are developed according to clutter models and logic used to estimate the unknown clutter parameters. Some examples are CA-CFAR [1], ML-CFAR [2], OS-CFAR [3, 4], OSGO and OSSO [5]. Barkat and Varshney [6] extended the CFAR technique to the distributed detection. They analyzed the CA-CFAR detection with multiple sensors and fusion center. Namely, each CA-CFAR detector transmits a binary decision to the fusion center where a final decision is made based on AND or OR rule. Distributed OS-CFAR detectors with the AND or the OR fusion rules is considered by Uner and Varshney [7]. Amirmehrabi and Viswanathan [8] proposed a scheme called S+OS based on the local test statistics, which instead of the binary decision; each sensor transmits the sample from the test cell and a designated order statistics from the available set of the reference observations surrounding the test cell to the fusion center. In [9], a new distributed CFAR test called the normalized test statistics (NTS) was proposed. In this test, the sum of local test statistics is compared with a threshold to decide the presence/absence of a target. It was shown in [9] that NTS provides a considerable performance gain over OR and AND fusion rules under the condition of large signal-to-clutter power ratio and moderate shape parameter values of the Weibull clutter. The reference [9] also proposed three types of distributed CFAR detection based on local test statistics; they are called R (ratio), S (substrate) and P (pulse) schemes. Distributed detection of signals in non-Gaussian clutter is also considered in [10]. However, it is necessary to find more effective local processing measures and effective fusion rules in fusion center to improve the distributed detection. In [11], Hammoudi and Soltani considered the problem of distributed CA-CFAR and OS-CFAR detections using fuzzy spaces and fuzzy fusion rules in Gaussian clutter.

In this paper, two general types of distributed detectors based on binary and fuzzy weighting function in fusion center are developed. In the binary type, the receiver data in the fusion center weighted according to a binary weighting function, whereas, in the fuzzy type, it is fuzzy weighting function. In the binary and fuzzy types, we analysed various distributed detection schemes based on ML (OS) CFAR processor in local detectors and maximum, minimum, summation and algebraic product, algebraic sum, probabilistic OR and Lukasiewicz t-conorm fuzzy rules in fusion center.

II. PROBLEM FORMULATION

Consider a two-sensor distributed network, Here \( x_i \) is the observation (excluding the test sample) and \( x_0 \) is the sample in the test cell, where \( i=1,2 \) indicates the number of the sensors, and \( j=1,2,...,N_i \) represents the sample number in the range cell available to the \( i^{th} \) sensor. It is assumed that both the sensors scan the same search environment and \( x_{i1}, x_{i2}, ..., x_{iN_i} \) are independent identically distributed (iid) random variables. Under the condition of Weibull envelope clutter, the outputs of square-law detector follow a Weibull distribution with probability density function (pdf) as:

\[
f_i(x) = \frac{c_i}{2b_i^2} \left( \frac{x}{b_i} \right)^{c_i-1} \exp \left(-\left(\frac{x}{b_i}\right)^{c_i}\right)
\]

(1)

Here, \( b_i^2 \) is the scale parameter and \( 0.5c_i \) is the shape parameter of the \( i^{th} \) sensor. In the Weibull clutter, when shape parameter is known, the detection is performed as

\[
\frac{x_{i0}}{b_i^2} > g_i
\]

(2)

where \( b_i^2 \) is the estimation of the scale parameters which can be formed by CFAR algorithm such as Maximum Likelihood (ML) [2] or Order Statistics (OS)[4]. The \( g_i \) is also a complicated function of desired \( P_{FA} \), the number of reference samples and the method used for parameter estimation. In this...
paper, we consider the situation in which the sensors locally perform CFAR processing before transmitting data to the fusion center. The data received by the fusion center, in this situation is, \(s_i\), where

\[
s_i = \frac{x_{ci}}{b_i}, \quad i = 1, 2
\]

(3)

At the fusion center, it was assumed that first, the received data from local sensor \(s_i\), \(i=1,2\), are weighted according to weighting function and then are combined by deterministic rules to construct global test statistics. Thus, the global test statistics in the fusion center is made as follows

\[
\begin{align*}
G(w(s_i), w(s_j)) &> T, \quad H_1 \text{ is true} \\
G(w(s_i), w(s_j)) &< T, \quad H_0 \text{ is true}
\end{align*}
\]

(4)

where operator \(w(\cdot)\) is weighting function in fusion center and operator \(G(\cdot)\) is the global test statistics which is generated from \(w(s_i), i=1,2\), by a predetermined fusion rule. Finally, the statistics \(G\) is compared with threshold \(T\) to decide about presence or absence of target, i.e. \(H_1\) or \(H_0\) hypothesis, respectively. In this paper, we consider binary and fuzzy weighting function, \(w(\cdot)\), and use several fusion rules such as an summation, algebraic product, algebraic sum, probabilistic OR and Lukasiewicz t-conorm to produce global test statistics.

### A. Binary Distributed Detectors

In this subsection, we adapt the structure of distributed detectors such as Max, min and NTS in [9] based on global test statistics in eqn.4 and summation fusion rule. These detectors, Max, min and NTS, are defined as [9]:

- **Max:**

  \[
  \max(s_1, s_j) > T, \quad s_i \leq T
  \]

  (5-a)

- **min:**

  \[
  \min(s_1, s_j) > T, \quad s_i < T
  \]

  (5-b)

- **NTS (Sum):**

  \[
  s_1 + s_j > T, \quad s_i < T
  \]

  (5-c)

By considering the global test statistics in Max, min and NTS as:

\[
G(w(s_i), w(s_j)) = w(s_i) + w(s_j) = \alpha s_i + \beta k_i
\]

(6)

The weighting coefficient \(\alpha\) and \(\beta\) in max, min and NTS, can be obtained to be

\[
\begin{align*}
\text{Max}: & \quad \alpha = 1, \beta = 0 \quad \text{if} \quad s_i \geq s_j \\
& \quad \alpha = 0, \beta = 1 \quad \text{if} \quad s_i < s_j
\end{align*}
\]

(7-a)

\[
\begin{align*}
\text{min}: & \quad \alpha = 0, \beta = 1 \quad \text{if} \quad s_i \leq s_j \\
& \quad \alpha = 1, \beta = 0 \quad \text{if} \quad s_i > s_j
\end{align*}
\]

(7-b)

NTS: \(\{\alpha = 1, \beta = 1\}\)

Therefore, in these distributed detectors, based on the new construction (eqn.4), it was found that, firstly the received data from local sensors \(s_i, i=1,2\), are weighted according to binary weighting function, \(\alpha\) and \(\beta\), and then are combined by summation rule to construct the global test statistics. Generally, such binary weighting function causes significant loss of information in fusion center. In order to reduce this imperfection, in the following subsection, we introduce two soft weighing functions based on ML and OS-CFAR detectors.

In the following, based on local CFAR algorithm ML (OS), we named the distributed detectors based on (6) and (7) as B-ML-M (B-OS-M), B-ML-m (B-OS-m) and B-ML-S (B-OS-S), respectively. The abbreviations denote, in order, type of weighting function (binary (B) or fuzzy (F)), CFAR algorithm used in local detectors (ML or OS), and fusion rules to construct general test statistics (Max (M), min (m), NTS (S)).

### B. Fuzzy Distributed Detectors

As mentioned in the previous subsection, employing binary weighting function in fusion center certainly causes significant loss of information in the fusion center. Recently, some authors have used fuzzy logic in detection to improve the system performance [11-13]. Indeed, in the fuzzy logic, binary functions are replaced with the soft functions. In this paper, we employ the soft weighing function, \(w(s_i)\) which can be implemented as a fuzzy membership function, assigning membership to the hypothesis \(H_0\). To evaluate weighting function, we use uppercase letters to denote random variables and lowercase letters to denote their corresponding observation. The weighing function, \(w(s_i)\), is defined so that the weighted values are distributed uniformly on \([0,1]\) under \(H_0\), as [12]:

\[
w(s_i) = \int_{s_i} f_{s_i}(z | H_0)dz = 1 - F_{s_i}(s_i | H_0)
\]

(8)

where, \(f_{s_i}(s_i | H_0)\) is the pdf of random variable \(S_i\) under hypothesis \(H_0\) and can be expressed as [14, pp.187]:

\[
f_{s_i}(s_i | H_0) = \int f_{x_{ci}}(s_i, y | H_0) f_y(y | H_0)dy
\]

(9)

In the following section, we obtain weighting function based on ML or OS as local CFAR algorithm and use four fuzzy rules such as algebraic product, algebraic sum, probabilistic OR and Lukasiewicz t-conorm to construct the global test statistics.

1) **Weighting function of ML in Local Sensors:** The ML-CFAR algorithm uses maximum likelihood estimation of \(b_i^2\) to reach CFAR property. When \(c_i\) is assumed to be known and square law detector is employed, \(b_i^2\) can be obtained as [2]:

\[
b_i^2 = \left( \frac{1}{N} \sum_{j=1}^{N} x_{ij}^2 \right)^{\frac{1}{2}}
\]

(10)
Thus, the weighing function of ML-CFAR can be obtained as [13]:

\[
w(s_i) = \left(1 + \frac{\sqrt{s_i}}{N_i}\right)^{-n_i}
\]  \hspace{1cm} (11)

2) Weighting function of OS in Local Sensors: In the OS algorithm, the observation samples, \(x_i\), are first rank ordered according to increasing magnitude. This rank ordered sequence are denoted by \(x_{(1)}, x_{(2)}, \ldots, x_{(N_i)}\) where \(x_{(i)}\) denote the \(i^{th}\) largest order statistics of \(x_1, x_2, \ldots, x_{N_i}\). From this sequence, \(k_i^{th}\) ordered value, \(x_{(k_i)}\), is selected as estimation of scale parameter. In this case, the weighting function \(w(s_i)\) is obtained to be [13]:

\[
w(s_i) = \prod_{i=1}^{N_i} \left(1 + \frac{\sqrt{s_i}}{N_i - i}\right)^{-1}
\]  \hspace{1cm} (12)

In the following, we use fusion rules such as Algebraic product (AP) and Algebraic sum (AS), probabilistic OR (PO) and Lukasiewicz t-conorm (LTC) fusion to construct test global statistics. In the following the relationship between probability of false alarm (PFA) and detection threshold, T, based on the mentioned fusion rules are derived.

1) Algebraic product (AP): The algebraic product fuzzy fusion rule in fusion center is product of \(w(s_1)\) and \(w(s_2)\) as [15]:

\[
G = w(s_1) \times w(s_2)
\]  \hspace{1cm} (13)

2) Algebraic sum (AS): The algebraic sum fuzzy fusion rule in fusion center is defined by [16]:

\[
G = w(s_1) + w(s_2)
\]  \hspace{1cm} (14)

3) Probabilistic OR (PO): The probabilistic OR (sum) fuzzy fusion rule in fusion center is defined by the relation [15]:

\[
G = w(s_1) + w(s_2) - w(s_1) \times w(s_2)
\]  \hspace{1cm} (15)

4) Lukasiewicz t-conorm (LTC): The Lukasiewicz t-conorm fuzzy fusion rule in fusion center is defined by the relation [16]:

\[
G = \min\left(w(s_1) + w(s_2), 1\right)
\]  \hspace{1cm} (16)

From eqn.8, it is clear that \(w(s_i)\) is a decreasing function as CDF function \(F_s(s_i | H_1)\) is an increasing function. On the other hand, Since random variable of \(w(s_1)\) and \(w(s_2)\) are also uniformly distributed in \([0, 1]\), it is easy to prove that \(G\) in eqns.13-16 is also a decreasing function of \(s_i\). Thus, the global test statistics in fusion center can be written as [13]:

\[
\begin{aligned}
G \leq T, & \quad H_1 \text{ is true} \\
G > T, & \quad H_0 \text{ is true}
\end{aligned}
\]  \hspace{1cm} (17)

Based on eqn.17, the \(P_{FA}\) in the fusion centre can be represented as:

\[
P_{FA} = P(G \leq T | H_0) = F_G(T)
\]  \hspace{1cm} (18)

After solving eqn.18 for algebraic product [11], algebraic sum [11], probabilistic OR [11] and Lukasiewicz t-conorm fuzzy rules [13], the relationship between \(P_{FA}\) and \(T\) can be obtained, respectively, as:

\[
P_{FA, AP} = T \left(1 - \ln(T) \right)
\]  \hspace{1cm} (19)

PFA,PO = T + (1-T)\ln(1-T)
\]  \hspace{1cm} (20)

PFA,PO = T + (1-T)\ln(1-T)
\]  \hspace{1cm} (21)

PFA,LTC = F_G(T) = 0.5T^2
\]  \hspace{1cm} (22)

In the following based on local CFAR algorithm ML (OS) and fuzzy (F) weighting function in fusion center, the resulting detectors are named as F-ML-AP (F-OS-AP), F-ML-AS (F-OS-AS), F-ML-PO (F-OS-PO) and F-ML-LTC (F-OS-LTC).

III. PERFORMANCE STUDY

In the simulations of this section, as many similar works, we consider the Rayleigh model for fluctuating targets, representing Swerling I (SWI) and Swerling II (SWII) (single pulse assumption) and we examine and compare the \(P_D\) and \(P_{FA}\) performance of the mentioned distributed detector with each other. Unless otherwise stated, we consider the symmetrical case where all the parameters of the local detectors are assumed to be the same, i.e., \(N_1=N_2=N=24\), \(C=c_1=c_2\), \(SCR_1=SCR_2=SCR\) and \(k_1=k_2=k=20\) (OS-CFAR local detectors case).

In Fig.1, the probabilities of detection \(P_D\) against the signal-to-clutter ratio \(SCR\) of the F-ML-AP, F-ML-PO, F-ML-LTC, F-ML-AS, B-ML-M, B-ML-m and B-ML-S distributed detectors are shown. The superior performance of the F-ML-AP detector over the B-ML-S, B-ML-M, F-ML-PO, F-ML-LTC, F-ML-AS and B-ML-m can be observed. Fig.2 shows the detection performance of the distributed system where the CFAR algorithm in local detectors are OS. The same observation as in Fig.1 can be made.

The effect of unequal received SCR in two sensors on the detection performance was evaluated in the Fig.3. In Fig.3, the SCR at sensor 1 is \(SCR_1=20dB\), the SCR in sensor 2 \(SCR_2\) range from -20dB to 40dB. For \(SCR_2\) from -20dB through 0, the better performance of the B-ML-M detector followed by the B-ML-S, over the F-ML-AP, F-ML-PO, F-ML-LTC, F-ML-AS, B-ML-M and B-ML-m can be observed. But, for \(SCR_2>0dB\), this figure demonstrate superiority of F-ML-AP over all others.

In the Fig.4, we studied the effect of presence of interfering targets in reference window of two detectors on the detection performance of OS distributed detector. In this figure, the number of interfering targets at sensor 1 is \(b_1=2\), the number of interfering targets in sensor 2 \(b_2\) ranges from 0 to 10. From Fig.4, it is seen for \(b_2\) from 0 through 4, the F-OS-AP detector performs better than F-OS-PO, F-OS-LTC, F-OS-AS, B-OS-m, B-OS-S and B-OS-M CFAR detectors. But, for \(b_2\) greater than 4 a significant degradation in detection performance occurs for these detectors. This expected, because the maximum number of tolerable interfering targets in OS based algorithm depends on the selected rank and in this case equals to 4 \((N-k=4)\). In Fig.4, it can also be seen that for \(b_2>8\), the F-OS-AP has lower \(P_D\) as compared with B-OS-M and B-OS-S, but has a much better performance than the other detectors.
IV. CONCLUSIONS

In this paper, we have developed two types of distributed CFAR detector using distributed sensors in Weibull clutter. We conclude from the study of a two-sensor system that for the homogenous situation Weibull clutter, the detection performance of the F-ML-AP is considerably better than that of the all other distributed detectors. When the two sensors don’t see the same SCR (i.e. SCR₁ ≠ SCR₂), the simulation results have also indicated robust and superior performance of F-ML-AP (F-OS-AP) detector. In multiple targets situation, the results have demonstrated better performance of distributed detector based on OS CFAR algorithm to the distributed detectors based on ML CFAR algorithm.

REFERENCES


Fig.1 PD versus SCR performance of distributed detector based on ML CFAR algorithm in local sensors when C= 0.8

Fig.2 PD versus SCR performance of distributed detector based on OS CFAR algorithm in local sensors when C= 0.8

Fig. 3 PD versus SCR₂ performance of distributed detector based on ML CFAR algorithm in local sensors when SCR₁=20dB

Fig.4 detection performance of distributed detector based on OS CFAR algorithm versus number b₂ of interfering target at sensor 2 when b₁=2