Fast Computation of Radiation Fields of Reflector Antennas by FFT Method

Z.H. Firouzeh* (1), A. Zeidaabadi-Nezhad (1), and H. MirMohammad-Sadeghi (1)
(1) Information and Communication Technology Institute (ICTI), Isfahan University of Technology (IUT), 84154 Isfahan, Iran, www.icti.ir

Abstract:

The rapid calculation of the far-field radiation patterns of a reflector antenna fed by a feed in an arbitrary location is reported. The reflector can be the intersection of a cone with any surface of revolution or an offset sector of any surface of revolution. The feed is assumed to be linearly polarized and located in a desired place in front of the reflector. The reflector is analyzed by Geometrical Optics (GO) theory and Physical Optics (PO) approximation. Then, radiation fields are calculated by aperture distribution method in which the radiation integrals computed fast by FFT method. The program based on MATLAB can analyze and simulate all shaped reflector antennas with large dimensions in regard to the wavelength. For example, a shaped reflector antenna fed by two displaced feed elements has been simulated and compared to FEKO results. The presented numerical method can be applied to all the shaped reflector antennas fed by defocused feeds to simulate radiation characteristics fast and accurately.

Keywords:
Reflector Antenna, Radiation Pattern, FFT, Aperture Distribution Method

1- Introduction

For satellite communications where low SLLs are essential to achieve, the offset fed reflector antenna can be used with good isolation between adjacent high gain beams operated over the same frequency band. The feeds may or may not be on focus depending upon the particular configuration. A cluster of feeds can be used in the focal region of offset reflectors for multiple beams application [1, 5]. In defocused feed reflector antennas, it should be regarded some effects such as cross polarization, coma, and astigmatism to avoid degradation of the antenna efficiency.

There have been published lots of papers to analyze the reflector antennas analytically due to calculate the antenna parameters and radiation fields [2], but nowadays by use of computers the numerical techniques can be applied to the reflector antennas due to simulate fast with good accuracy.

To calculate the radiation patterns, the equations of geometrical optics are used to calculate the reflected electric field using the radiation patterns of the feed and the parameters defining the reflector surface. Also the direction of the reflected ray and the point of intersection of the reflected ray with the aperture plane are obtained by use of geometrical optics. These fields comprise the aperture distribution which is integrated over the aperture plane by FFT method to yield the far-field radiation pattern and to calculate other antenna parameters.

A feed at the focal point of a parabola forms a beam parallel to the focal axis. Thus, "the project rays strike the aperture plane at points defining radial lines emanating from the origin. For other feed positions these data points will not lie on constant coordinate lines in any convenient coordinate system. Moreover, the point configuration changes each time the feed position and/or reflector geometry is changed" [5]. In this paper an algorithm is presented to evaluate aperture numerical integration by FFT method. This coordinate system is used for all antenna configurations. The preceding algorithm can be applied to all shaped reflector antennas which has been illuminated by defocused feeds with arbitrary patterns.

The program based on MATLAB can analyze and simulate all shaped reflector antennas with large dimensions in regard to the wavelength. For example, a shaped reflector antenna fed by two displaced feed elements has been simulated and compared to FEKO results. The presented
numerical method can be applied to all the shaped reflector antennas fed by defocused feeds to simulate radiation characteristics fast and accurately.

2- Analysis theory of reflector antennas

The three-dimensional geometry of a paraboloidal reflector system has been shown in Fig. 1. The projected cross-sectional area of reflector on the aperture plane -the opening of the reflector- is $S_0$, and on the focal plane is $S'_0$. The equations of geometrical optics are used to calculate the reflected electric field using the radiation patterns of the feed and the parameters defining the reflector surface. Also the direction of the reflected ray is obtained by use of geometrical optics theory. By use of Physical Optics (PO) approximation, the aperture distribution method has been applied to parabola antenna to yield the radiation characteristics of the antenna [3, 4].

The equations here are developed for a paraboloid of revolution or any offset sector of a paraboloid of revolution. The radiation integrals over $S'_0$ to compute the far fields can be written as [3]:

$$E_{\phi S} = \frac{j \beta e^{-j \beta r}}{4 \pi r} \left(1 - \cos \theta \right) \iint_{S_0} \left( - E_{ax} \cos \phi - E_{ay} \sin \phi \right) e^{j \beta (x' \sin \theta \cos \phi + y' \sin \theta \sin \phi)} dx' dy' \quad (1a)$$

$$E_{\theta S} = \frac{j \beta e^{-j \beta r}}{4 \pi r} \left(1 - \cos \theta \right) \iint_{S_0} \left( - E_{ax} \sin \phi + E_{ay} \cos \phi \right) e^{j \beta (x' \sin \theta \cos \phi + y' \sin \theta \sin \phi)} dx' dy' \quad (1b)$$

![Fig. 1: Three-dimensional geometry of a paraboloidal reflector system [3]](image)

$E_{ax}$ and $E_{ay}$ which are $x$- and $y$-component of the reflected fields over the focal plane. A feed at the focal point of a parabola forms a beam parallel to the focal axis. Consequently, the only difference between fields on $S_0$ and $S'_0$ is the constant phase because of the distance between the aperture plane $S_0$ and the focal plane $S'_0$. With the source off the focus, the reflected fields from the reflector are not parallel to the focal axis which results in a severe phase distortion between fields on $S_0$ and $S'_0$. Therefore, the integral equations (1a) and (1b) are calculated over the aperture plane $S_0$, not $S'_0$. These fields comprise the aperture distribution which is integrated over
the aperture plane by FFT method to yield the far-field radiation pattern and to calculate other antenna parameters.

3- Calculation of the radiation integrals by FFT method

Fig. 2: (a) Plane $S'_0$ is the projected cross-sectional area of reflector on the focal plane
(b) A rectangular mesh covering plane $S'_0$

A rectangular mesh covers the focal plane $S'_0$, and reflected fields from the reflector are calculated at the mesh points as shown in Fig. 2. By using Physical Optics (PO) approximation, the reflected fields out of $S'_0$ are vanished. Two-dimensional FFT method is used to compute the integral equations (1a) and (1b) rapidly. The mesh grid is generated as the following:

$$dx' = \frac{d}{M - 1} \quad x' = -\frac{d}{2} + m dx' \quad m = 0, 1, ..., M - 1$$

$$dy' = \frac{d}{N - 1} \quad y' = -\frac{d}{2} + n dy' \quad n = 0, 1, ..., N - 1$$

The relation between mesh points and $\theta$, $\phi$:

$$A = \frac{d}{\lambda} \frac{M}{M - 1} \quad , \quad k = -ASin\theta_{kl} Cos\phi_{kl} \quad , \quad k = 0, 1, 2, ..., M - 1$$

$$B = \frac{d}{\lambda} \frac{N}{N - 1} \quad , \quad l = -BSin\theta_{kl} Sin\phi_{kl} \quad , \quad l = 0, 1, 2, ..., N - 1$$

$$0 \leq \theta_{kl} \leq \pi \quad , \quad 0 \leq \phi_{kl} \leq 2\pi \text{ or } -\pi \leq \phi_{kl} \leq \pi$$

$$\theta_{kl} = \sin^{-1}\left(\left[\left(\frac{k}{-A}\right)^2 + \left(\frac{l}{-B}\right)^2\right]^{1/2}\right)$$

$$\phi_{kl} = \tan^{-1}\left(\frac{-Al}{-Bk}\right)$$

Finally, radiation fields of $E_{\theta S}$ and $E_{\phi S}$ are computed based on FFT2. It can be written as:

$$P_x = \exp\left(-j\beta d\frac{d}{2}\right)\exp\left(-j\beta y\frac{d}{2}\right) dx' dy' FFT2(E_{ax})$$

$$P_y = \exp\left(-j\beta x\frac{d}{2}\right)\exp\left(-j\beta y\frac{d}{2}\right) dx' dy' FFT2(E_{ay})$$

(4a)
\[ E_{\theta} = \frac{\beta e^{-jkr}}{4\pi r} \left( 1 - \cos \theta_{kl} \right) \left( -\cos \phi_{kl} P_x - \sin \phi_{kl} P_y \right) \]
\[ E_{\phi} = \frac{\beta e^{-jkr}}{4\pi r} \left( 1 - \cos \theta_{kl} \right) \left( -\sin \phi_{kl} P_x + \cos \phi_{kl} P_y \right) \]

It is noticeable that real angles of \( \theta_{kl} \) which have been found by the equations of (3) can be used. Negative values of l and k will be used to find the fields in other regions. In addition, number of main points of the major lobe found by FFT method is constrained. Therefore, it is interpolated from above points to obtain E- and H-planes half-power beamwidths (HP_E and HP_H).

To find E- and H-plane radiation patterns, it should be \( \varphi = \frac{\pi}{2} \) corresponding axis v, and \( \varphi = 0 \) corresponding axis u. This method can be applied to all shaped reflector antennas with defocused feed elements provided that dimensions of the reflector are large in regard to the wavelength.

4- Analysis of a shaped reflector antenna fed by two displaced feed horns

Several reflector antennas have been simulated by the presented method and the results have a good agreement with measurements. For example, a shaped reflector antenna fed by two displaced feed horns has been simulated. The operating frequency of the antenna is 1.4 GHz. Reflector Aperture is 7.0m high by 13.5m wide with focal axis of 5.31m. The profile of azimuth curve is parabolic and the profile of elevation curve is an unusual function. The 3D mathematical function which determines the reflector surface of the antenna is:

\[ z = -0.0471 x^2 + 5.3100 \cos 0.6364 \left( \frac{y}{4.9267} \right) \]

Fig. 3: A shaped reflector antenna with two feed horns

The reflector is fed by two feed horns which have been displaced from each other 0.37m in y-direction symmetrically in relation to the origin. The feed horn which has been located in (0,0,-0.185) radiates high beam and the other located in (0,0,0.185) radiates low beam.

Simulated results by MATLAB program has been shown in Table 1. Both of M and N for meshing the reflector aperture are 512. The collapsed time of simulation by MATLAB program is about one-half of FEKO simulation time. FEKO results have been given in Table 2. Radiation patterns of the antenna have been measured by open-site method because of its large dimensions.
The gain of high beam is 35.5dB and that of low beam is 34.5dB. Azimuth half power beamwidth (HPH°) is 1.2° and elevation HPBW (HPH°) is about 10.5°. The presented numerical method has a good agreement with measurements and has a high speed for simulation in comparison to FEKO software.

Table 1: Radiation characteristics simulated by MATLAB program

<table>
<thead>
<tr>
<th>Beam</th>
<th>Y (m)</th>
<th>Elevation. (degree)</th>
<th>Azimuth. (degree)</th>
<th>Gain (dB)</th>
<th>HP_H°</th>
<th>HP_E°</th>
<th>SLL_E (dB)</th>
<th>SLL_H (dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low</td>
<td>+0.185</td>
<td>-1.85</td>
<td>+90</td>
<td>34.1</td>
<td>11.1</td>
<td>1.0</td>
<td>40</td>
<td>35</td>
</tr>
<tr>
<td>High</td>
<td>-0.185</td>
<td>+1.85</td>
<td>+90</td>
<td>34.1</td>
<td>11.1</td>
<td>1.0</td>
<td>40</td>
<td>35</td>
</tr>
</tbody>
</table>

Table 2: Radiation characteristics simulated by FEKO software

<table>
<thead>
<tr>
<th>Beam</th>
<th>Y (m)</th>
<th>Elevation (degree)</th>
<th>Azimuth (degree)</th>
<th>Gain (dB)</th>
<th>HP_H°</th>
<th>HP_E°</th>
<th>SLL_E (dB)</th>
<th>SLL_XOZ (dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>High</td>
<td>-0.185</td>
<td>+2</td>
<td>+90</td>
<td>33.4</td>
<td>12.2</td>
<td>0.6</td>
<td>34</td>
<td>32</td>
</tr>
<tr>
<td>Low</td>
<td>0.185</td>
<td>-2</td>
<td>+90</td>
<td>33.4</td>
<td>12.2</td>
<td>0.6</td>
<td>34</td>
<td>32</td>
</tr>
</tbody>
</table>

5- Conclusion

The development and application of a numerical technique for the rapid calculation of the far-field radiation patterns of a reflector antenna fed by defocused feeds have been reported. The reflector is analyzed by Geometrical Optics (GO) theory and Physical Optics (PO) approximation. Aperture distribution method has been used to calculate radiation fields and fast computation of the radiation integrals realized by FFT method. By using the presented numerical method, the program based on MATLAB can analyze and simulate all shaped reflector antennas with large dimensions in regard to the wavelength. For example, a shaped reflector antenna fed by two displaced feed horns has been simulated and compared to FEKO results. This numerical method can be applied to all the shaped reflector antennas fed by defocused feeds to simulate radiation characteristics fast and accurately.

References: